LIMITED CATHODE PROCESSES UNDER CONDITIONS OF A POWERFUL PULSE DISCHARGE

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We examine the thermophysical processes at the initial and final stages of a pulse through joint consideration of surface evaporation and thermal field emission as cooling effects and the effects of all heat sources acting on the cathode.

In comparison with the anode, the role of the cathode in the development and maintenance of discharge is more significant. Functioning as a collector of positive ions, the cathode at the same time appears as a powerful emitter of electrons, as a consequence of which we have a two-component current in the zone of the cathode potential drop ($j_c = j_e + j_i$). The space discharge of the latter, as demonstrated by Mackeown [1] excites an electric field at the cathode surface, i.e.,

$$E^2 = 7.57 \cdot 10^5 U_c^{1/2} \left[(\psi/\psi_0) - 1\right] j_e, \tag{1}$$

where ψ and ψ_0 denote j_i / j_e and $(m_e / m_i)^{1/2}$.

In the initial stage of the pulse, when the cathode temperature remains quite low, the cathode spot under the action of the cited field may function only as a cold emitter. This means that the density of the electron component of the cathode current must be subject, in this case, to the Nordheim–Fowler equation [2], i.e.,

$$j_e = (1.55 \cdot 10^{-6} E^2/\varphi) \exp\left[-6.85 \cdot 10^7 \varphi^{3/2} v(y)/E\right], \tag{2}$$

where the dimensionless function v(y), correct to 0.1%, may be approximated – as assumed by Andreev [3] – by the expression

$$v(y) = 0.965 - (1.05 \cdot 10^{-7} E/\varphi^2). \tag{3}$$

(The quantity j_e, U_c, E, and φ in (1)-(3) must be expressed, respectively, A/cm^2, V, V/cm, and eV.)

Equations (1) and (2) include the three quantities E, ψ, and j_e which are functions of the pulse regime, or what is the same, the equations include E, ψ, and j_c, where $j_c = (1 + \psi) j_e$. Eliminating E from (1)-(3), we can derive a separate equation for the two remaining quantities:

$$j_c = A(1 + \psi)/\left[(\psi/\psi_0) - 1\right] \{B + \ln\left[(\psi/\psi_0) - 1\right]\}^2, \tag{4}$$

where the constants $A = 5.80 \cdot 10^9 \varphi^3 / U_c^{1/2}$ and $B = 0.157 + (7.2/\varphi^{1/2}) + \ln(U_c^{1/2}/\varphi)$ have individual values for the various metals. Data on these metals are presented in Table 1, on the basis of the experimental estimates of U_c, based on the work Kesaev [4].

The results from the solution of (4) and (1) are shown in Fig. 1, which illustrates the functional relationship between E, ψ, and j_c over a rather broad range of pulse regimes. As we can see from the figure, with a change in the density of the cathode spot the coefficient ψ changes within extremely broad limits. Since it determines the relative value of the ion current which transmits the thermal surface effect to the cathode, it is clear that these variations in the coefficient will be markedly reflected in the thermophysical process of the cathode at the initial stage of the pulse.

The density of the ion heat flow transmitted to the cathode is determined [5] from the expression

$$F_i = \mu(1 + \eta + \nu) U_c j_i = \left[\psi/(1 + \psi)\right] \mu(1 + \eta + \nu) U_c j_c \tag{5}$$

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TABLE 1. Values of the Constant A, B, and ψ_0 for Certain Metals

Metals	Cd	Zn	Al	Cu	W
A	$9{,}32\cdot10^{10}$	$7{,}75\cdot10^{10}$	$6{,}32\cdot10^{10}$	$9{,}27\cdot10^{10}$	$1{,}31\cdot10^{11}$
B	3,75	3,91	4,11	3,76	3,44
ψ_0	$2{,}25\cdot10^{-3}$	$2{,}92\cdot10^{-3}$	$4{,}48\cdot10^{-3}$	$2{,}96\cdot10^{-3}$	$1{,}73\cdot10^{-3}$

where the sum of the relative coefficients $\eta = 2kT_i/eU_c$ and $\nu = U_i/U_c$ assumes a value no larger than 0.3-0.5, while the expression $\mu(1+\eta+\nu)$ ranges within the limits of 1.2 to 1.5 for the various metals.

With a one-dimensional approximation of the thermophysical process for a semibounded cathode ($0 \le x' \le \infty$) the thermophysical problem at the initial stage of the pulse can be formulated, as follows from [6], in the following manner:

$$\frac{\partial T(x', t)}{\partial t} = a\frac{\partial^2 T(x', t)}{\partial x'^2} + bu j_c \frac{\partial T(x', t)}{\partial x'} + u j_c^2$$

$$-\lambda\frac{\partial T(0, t)}{\partial x'} = [\psi/(1+\psi)]\,\mu(1+\eta+\nu)U_c j_c. \tag{6}$$

$$\frac{\partial T(\infty, t)}{\partial x'} = 0; \quad T(x', 0) = T_0, \tag{7}$$

in which we have taken into consideration all of the heat sources: the surface source, and the Lenz–Joule and Thomson volume sources. Here the quantities $a = \lambda/c_V$, $u = \rho/c_V$, and $b = \pi^2k^2\lambda/e\varepsilon_0 L_0$ can be treated as constants, for which we can limit ourselves to their average values in the temperature range for the initial pulse stage (up to the cathode melting point T_{mp}).

Problem (6)-(7) can be presented in dimensionless form

$$\frac{\partial\theta(\xi, \tau)}{\partial\tau} = \frac{\partial^2\theta(\xi, \tau)}{\partial\xi^2} + B\frac{\partial\theta(\xi, \tau)}{\partial\xi} + 1, \tag{8}$$

$$\frac{\partial\theta(0, \tau)}{\partial\xi} = -q; \quad \frac{\partial\theta(\infty, \tau)}{\partial\xi} = 0; \quad \theta(\xi, 0) = 0, \tag{9}$$

where

$$\xi = A_\xi j_c x'; \quad \tau = A_\tau j_c^2; \quad q = A_q[\psi/(1+\psi)]\,\mu(1+\eta+\nu)U_c$$
$$\theta(\xi, \tau) = [T(x', t) - T_0]/(T_{mp} - T_0); \quad A_\xi = [u/a(T_{mp} - T_0)]^{1/2};$$
$$A_q = [a/\lambda^2 u(T_{mp} - T_0)]^{1/2}; \quad B = A_\xi b(T_{mp} - T_0).$$

The values of the individual characteristics A_ξ, A_τ, A_q, and B for the metals are given in [6]. Here we will point out only that for the various metals A_ξ varies from 10^{-7} to 10^{-6} m/A; A_τ varies from 10^{-18} to 10^{-16} $m^4/A^2\cdot sec$; A_q varies from 1 to 10 V^{-1}; and, finally, the dimensionless coefficient B varies from 10^{-2} to 10^{-1}.

Together with (8), it is a good idea to consider yet another two equations for the same boundary and initial conditions (9), i.e.,

$$\frac{\partial\theta(\xi, \tau)}{\partial\tau} = \frac{\partial^2\theta(\xi, \tau)}{\partial\xi^2} + 1, \tag{10}$$

$$\frac{\partial\theta(\xi, \tau)}{\partial\tau} = \frac{\partial^2\theta(\xi, \tau)}{\partial\xi^2}, \tag{11}$$

in the first of which the Thomson heat source has been eliminated, while the Thomson and Lenz–Joule heat sources have been eliminated in the second.

Problem (11), (9) determines the temperature field which arises at the cathode under the exclusive action of the surface heat source. The temperature at the cathode surface, according to this problem, is determined by the equation

$$\theta(0, \tau) = 2q\tau^{1/2}/\pi^{1/2}, \tag{12}$$

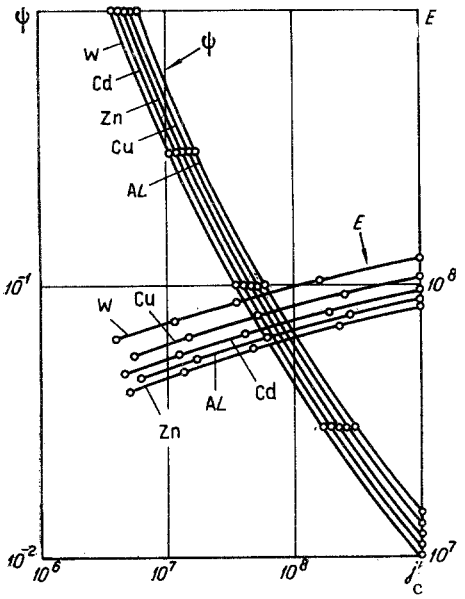

Fig. 1. Functional relationships between E (V/cm), ψ, and j_c (A/cm^2) for certain metals when the catbode is operating in the field-emission regime.

from which we find the instant of time τ_0' at which we attain the melting point $\theta(0, \tau_0') = 1$:

$$\tau_0' = \pi/4q^2. \tag{13}$$

This time can assume values of $\tau_0' > 1$ only when $q \ll \pi^{1/2}$, which is possible only in the region of extremely high current densities, when $j_c \gtrsim 10^8$ A/cm^2. Ahead of this region we have $\tau_0' \ll 1$.

Problem (10), (9) determines the temperature field at the cathode, produced as a consequence of the combined effect of surface heat sources and the Lenz–Joule volume heat sources, with the Thomson heat source eliminated. The temperature at the cathode surface, according to this problem, is expressed by the equation

$$\theta(0, \tau) = (2q\tau^{1/2}/\pi^{1/2}) + \tau. \tag{14}$$

If the process is considered at the instant τ_0' in (13), the difference between the temperatures in (14) and (12)

$$\Delta\theta_1(0, \tau_0') = \tau_0' \tag{15}$$

determines the additional development of the temperature generated at the instant τ_0' by the Lenz–Joule heat source. In the region of those pulse regimes in which $\tau_0' \ll 1$, we have $\Delta\theta_1(0, \tau_0) \ll 1$. In this case, the additional rise in temperature is quite insignificant, which points up the ineffectiveness of the Lenz–Joule heat source. However, in the region of high current densities $\tau_0' \gtrsim 1$, which means that $\Delta\theta_1(0, \tau_0') \gtrsim 1$. Under these conditions, the effectiveness of this heat source becomes significant.

Instead of the instant τ_0', for the following we should isolate the instant τ_0 at which the melting point $\theta(0, \tau) = 1$ at the cathode surface is attained as a result of the combined effect of the surface and Lenz–Joule heat sources. From (14) we find

$$\tau_0 = 1 + (2q^2/\pi)\left\{1 - \left[1 + (\pi/q^2)\right]^{1/2}\right\}. \tag{16}$$

With $q > \pi^{1/2}$ Eq. (16) changes into (13), i.e., $\tau_0 = \tau_0'$. However, in the region of high current densities, when $q < \pi^{1/2}$, Eq. (16) does not coincide with (13), i.e., we have $\tau_0 < \tau_0'$.

Problem (8), (9) determines the temperature field at the cathode under the combined action of all three heat sources. The temperature at the cathode surface, according to this problem, is determined by the expression

$$\theta(0, \tau) = \frac{2q\sqrt{\tau}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n (B/2)^{2n} \tau^n}{n!\,(2n+1)} - \frac{B^2 q \sqrt{\tau^3}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n (B/2)^{2n} \tau^n}{n!\,(2n+1)(2n+3)} - \frac{Bq}{2}\tau + \tau. \tag{17}$$

Since $B \ll 1$, when $\tau \leq 1$ we can limit ourselves to the zeroth approximation

$$\theta(0, \tau) = (2q\tau^{1/2}/\pi^{1/2}) - (Bq\tau/2)\left[1 + (B\tau^{1/2}/3)\right] + \tau. \tag{18}$$

The difference between the temperatures of (18) and (14) at the instant τ_0 determines the effect of the Thomson heat source

$$\Delta\theta_2(0, \tau_0) = -(Bq\tau_0/2)\left[1 + (B\tau_0^{1/2}/3\pi^{1/2})\right] \tag{19}$$

on the surface temperature of the cathode, generated by the remaining heat sources. This is a negative effect; however, its significance is quite unimportant. Indeed, when $q > \pi^{1/2}$, we have $\tau_0 \ll 1$. It follows from (19) that $\Delta\theta_2(0, \tau_0) < 0.22B$, and since $B \ll 1$, we have $\Delta\theta_2(0, \tau_0) \ll 1$. When $q < \pi^{1/2}$, we also have $\tau_0 < 1$, which for $B \ll 1$, as before, yields $\Delta\theta_2(0, \tau_0) \ll 1$. All of this indicates the relatively weak effectiveness of the Thomson heat source, regardless of the pulse regime.

Thus, in the initial stage of the pulse, in a region of current densities that are not too high, the thermophysical process at the cathode is determined exclusively by the surface heat source. However, with

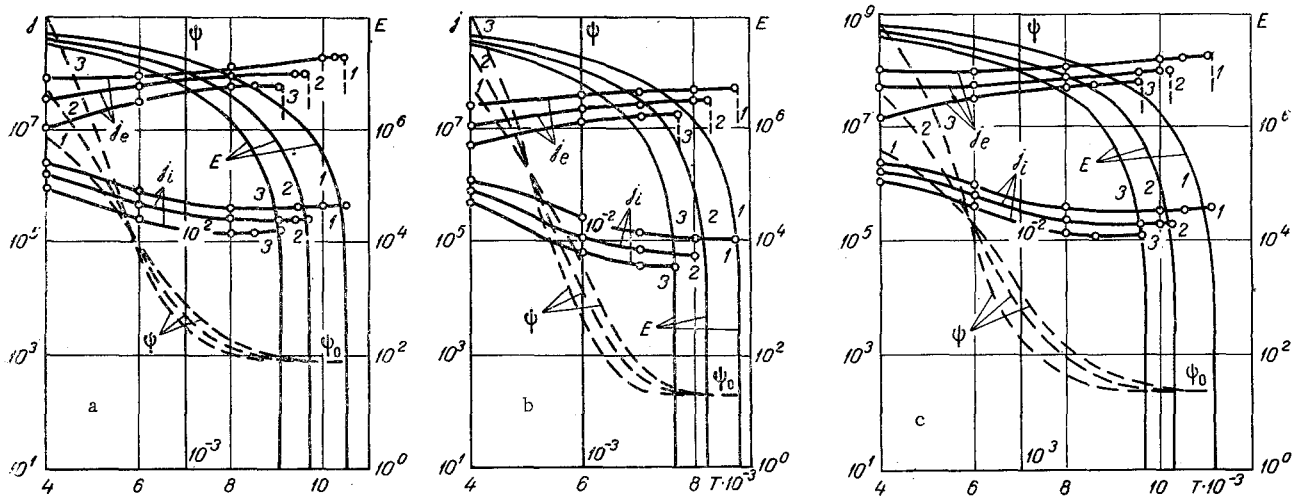

Fig. 2. Functional correspondence between the quantities j_e (A/cm^2), j_i (A/cm^2), ψ, E (V/cm), and T (°K) in the high-temperature region for a steady-state regime of a cathode spot of copper (a), tin (b), and silver (c): 1, 2, 3) values of h = 5, 10, and 20 μm.

transition to a region of high current densities, because of the pronounced drop in the coefficient ψ, conditions arise at the cathode under which the Lenz–Joule heat source may attain equivalent effectiveness – or even exceed the effectiveness – that is comparable to the surface heat source. This detail is an individual specific feature of the cathode alone, because no such phenomenon occurs at the anode [6]. As regards the Thomson heat source, the cathode does not provide conditions which lend themselves to its effective appearance.

On the basis of recent research, extremely high cathode current densities on the order of 10^7-10^8 A/cm^2 are characteristic [7, 8] of powerful pulse and arc discharge. Under these conditions, as follows from (16), the melting point is reached within 10^{-8}-10^{-9} sec. On conclusion of this initial phase, a further intensive rise in temperature is unavoidable, because there are no physical factors preventing this development of the process. It cannot be prevented by discrete migration of the cathode spot, since its retention at a single spot will substantially exceed the time indicated.

A further rise in temperature involves three newly generated specific features of the process: intensive surface evaporation; the penetration of the evaporation front into the depth of the cathode; and the change from field emission to thermal field emission. The power loss to evaporation and thermal field emission increases with a rise in temperature, which, in the final analysis, leads to a balance between these sources and the heat sources active at the cathode. In other words, with the passage of some period of intervening time, the thermophysical process changes into a steady-state regime.

If we eliminate the Thomson heat source from (6) and transform the latter to a movable coordinate system $x = x' - vt$ associated with the evaporation front, we can formulate a problem which yields a fully satisfactory one-dimensional approximation of the cathode thermophysical process which has gone over into a steady-state regime:

$$a\frac{d^2T(x)}{dx^2} + v\frac{dT(x)}{dx} + uj_c^2\exp(-\delta x) = 0, \tag{20}$$

$$-\frac{dT(0)}{dx} = (F_i - F_e - F_n)/ac_V; \quad \frac{dT(\infty)}{dx} = 0,$$
$$T(0) = T; \; T(\infty) = T_0, \tag{21}$$

where F_i is the ion heat flux (5), while F_e and F_n are the power losses as a result of thermal field emission and evaporation:

$$F_e = (\varphi - e^{3/2}E^{1/2} + 2kT)j_e/e;$$
$$F_n = vr_V; \tag{22}$$
$$v = v_0\exp(-T_m/T).$$

In (20) the quantity uj_c^2 represents the power of the volume source at the evaporation front, while $\exp(-\delta x)$ is a function approximating the power distribution from this source with respect to the coordinate. If we assume that $u = u_{mp}[1 + \alpha(T - T_{mp})]$ and $\delta = h^{-1}$, where h is the depth of the experimentally observed cathode microlunes, the approximation of this heat source w(x) in the form

$$w(x) = u_{mp}[1 + \alpha(T - T_{mp})]\, j_c^2 \exp(-\delta x) \tag{23}$$

can be regarded as quite reliable.

In the high-temperature region the radiation power losses $F_T \approx \sigma T^4$ attain very high values; however, they nevertheless remain several orders smaller than the losses due to evaporation and thermal field emission. Since $F_T \ll F_n$ and $F_T \ll F_e$, the radiation under boundary condition (21) is eliminated and not treated in the following.

The solution of (20)-(21) is known [5], and we will present it here in somewhat altered form:

$$[r_V + c_V(T - T_0)]\, v_0 \exp(-T_m/T)$$
$$= [\psi\mu(1 + \eta + \nu)U_c - (\varphi - e^{3/2}E^{1/2} + kT)/e]\, j_e + [(1 + \psi)^2 c_V u_{mp}/\delta][1 + \alpha(T - T_{mp})]\, j_e^2, \tag{24}$$

where the thermal field-emission current j_e in the temperature region under consideration is subject to the Murphy–Good equation [9, 10]

$$j_e = AT^2(\xi/\sin\xi)\exp z, \tag{25}$$

in which $\xi = 1.64 \cdot 10^{-2} E^{3/4}/T$; $z = -(1.16 \cdot 10^4 \varphi - 4.39E^{1/2})/T$; $A = 120$ A/cm$^2 \cdot$deg^2, if j_e, E, and φ are, respectively, expressed in A/cm^2, V/cm, and eV.

Equation (24) expresses the law of the conservation of energy for the steady-state regime that is applicable to the cathode in conjunction with two other equations – Eqs. (1) and (25) – the first of which determines the electric field at the cathode surface, while the second gives the thermal field emission. The system of three equations (1), (24), and (25) contains four quantities j_e, ψ, E, and T which are functions of the discharge regime. When reliable data are at hand with respect to any of these quantities, the three remaining quantities corresponding to the first can be found from the solution of the indicated system. However, the most recent information on any of these quantities is quite inadequate, particularly for a powerful pulse discharge.

Under these conditions, it is advisable to specify one of the quantities within rather broad limits and instead of a uniquely defined solution to derive a single-valued functional correspondence. From considerations of convenience, in this paper we have chosen – for this purpose – a temperature whose value is specified within the limits of applicability for (25); in this case, the system of Eqs. (1), (24), and (25), as a transcendental system and one that is particularly cumbersome for primitive calculations, has been solved for a number of metals on a computer.

The data shown by the curves correspond to a steady-state regime; when this state is reached, the evaporation intensity rises to such a high level that discharge in the cathode vapors not only becomes possible, but unavoidable. The formal condition for this process reduces to the fact that the yield factor $\gamma_{ni} = G_n/G_i$ should not be smaller than unity, i.e.,

$$\gamma_{ni} = (eG_0/j_i)\exp(-T_m/T). \tag{26}$$

Assuming $\gamma_{ni} = 1$, from this condition and the data in Fig. 2 we can determine the required temperature T_1, beginning from which discharge is possible in the cathode vapors when the cathode spot is in a steady state. With h = 5, 10, and 20 μm, the temperature T_1 for Ag is, respectively, equal to 4600, 4300, and 4200°K, it is equal to 4800, 4600, and 4400°K for Cu, and the corresponding figures for Sn are 5900, 5500, and 5200°K. However, for a powerful pulse discharge, when the removal of the vapor phase from the cathode zone is particularly extensive, the condition $\gamma_{ni} = 1$ is exceedingly inadequate. Under similar circumstances we should have the condition $\gamma_{ni} \gg 1$, which requires temperatures $T > T_1$. However, we do not know the extent to which the temperature region $T > T_1$ extends, nor whether or not it has an upper limit.

There are two possible approaches to the solution of the problem which has arisen here. First of all, we can assume that in the ionization space above the cathode spot there are conditions which impose some upper bounds on γ_{ni} and that this in turn will determine the upper limit of the temperature. However, it may also be assumed that in the case of a powerful pulse discharge the limit conditions will not appear

in the ionization space, but directly on the cathode spot itself. This standpoint, as shown by computer data, is based on more solid physical ground.

As can be seen from Fig. 2, with a rise in the temperature to some limit value $T = T_2$, the quantities ψ and E diminish to values of $\psi = \psi_0$ and $E \simeq 0$, whereas the thermal field emission current j_e rises without interruption to its upper limit and simultaneously changes into a thermoelectron current. The latter, when $T = T_2$, must break off because of the disappearance of the cathode potential drop ($U_c \simeq 0$), as well as because of the possibility of the formation of a barrier field, should the temperature T_2 be exceeded even slightly. (The values of T_2 in Fig. 2 correspond to the points of intersection for the curves of E on the temperature scale.)

The abrupt termination of the current when $T = T_2$ results in the unavoidable explosion of the cathode spot and its existence as an electron emitter. Such a final explosive effect is thermal in nature and associated with the volume superheating which is begun during the life of the cathode spot (grad T(0) > 0!) and then intensifies as the current comes to an end. The physical nature of such an explosive process is covered in detail in [11].

The existence of a powerful pulse discharge in the cathode vapors is thus possible only for a temperature range $T_1 \leq T \leq T_2$ bounded from below and from above. The density of the cathode current in this range of temperatures varies for the different metals (Fig. 2) between 10^7 and 10^8 A/cm^2, which does not contradict the experiment. At these current densities, the entry of the cathode spot into a steady-state regime occurs virtually instantaneously, while its transition from one steady state to another is virtually cophasal with a change in current density [12, 13].

The limited lifetime of the cathode spot is now known as an experimental fact. The observed upper limit regime which ends with the explosive effect explains this instability and thus determines the location of the pinch effect.

The role of the latter in the zone of the cathode potential drop becomes significant only under conditions in which the magnetic pressure $P_m = \mu_0 \mu H^2/2$ exceeds the gas-kinetic pressure $P_g = nkT$, i.e., when $\beta = (P_m/P_g) > 1$. Assuming that $H = I/2\pi r$, $n = (j_e/ev_e) = j_e(2m_e/e^3U_c)^{1/2}$, and carrying out the corresponding transformations, we find

$$\beta \simeq 4.2 \cdot 10^{-5} j_e r^2/T, \tag{27}$$

where the current density $j_e = j_c - j_i \simeq j_c$, the cathode-spot radius r, and its temperature T must be expressed, respectively, A/cm^2, μm, and °K. Having borrowed the data on the values of T for the copper cathode from Fig. 2a and the data on j_c and r from the experimental work [7], we find that the condition $\beta > 1$ is entirely possible for a current density lower than the limit. This means that the pinch effect can increase the current density to the limit level and to bring the cathode spot to the point of thermal explosion.

In the light of these concepts, the instability of the cathode spot is thermal in nature. It begins its existence with field emission, and ends its life with thermal field emission, evolving in the manner of an $E \rightarrow ET \rightarrow T$ emitter.

NOTATION

m_e, m_i	are the masses of the electron and the ion;
j_c, j_c, j_i	are the density of the cathode currents and its ion and electron components;
U_i, U_c	are the ionization potential and the cathode potential drop;
ε_0, φ	is the electron energy at the Fermi level and its work function;
k, L_0	are the Boltzmann constant and the Lorentz number;
T_i	is the ion temperature of the plasma in the zone of the cathode potential drop;
λ, ρ, v_0, c_V, r_V	are, respectively, the thermal conductivity, the resistivity, the speed of sound, the specific heat capacity at constant volume, and the specific volume heat of vapor formation for the cathode substance;
E, H	are the intensities of the electric and magnetic fields;
μ_0	is the magnetic constant of the vacuum.

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